THREE DIMENSIONAL GEOMETRY (3-D)

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 B

Given $2x^2 + 2y^2 + 2z^2 = 36$

$\Rightarrow x^2 + y^2 + z^2 = 18$

Distance from origin

$= \sqrt{x^2 + y^2 + z^2} = \sqrt{18} = 3\sqrt{2}$

Sol.2 C

$PA^2 - PB^2 = 2k^2$

$(x - 3)^2 + (y - 4)^2 + (z - 5)^2 - (x + 1)^2$

$- (y - 3)^2 - (z + 7)^2 - 2k^2$

$\Rightarrow 8x + 2y + 24z + 9 + 2k^2 = 0$

Sol.3 B

$\alpha + \beta = 90º$

$\alpha = 90 - \beta$

$\cos\alpha = \sin\beta$

$\cos^2\alpha = 1 - \cos^2\beta$

$\cos^2\alpha + \cos^2\beta = 1$ ....(1)

& $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\gamma = 0 \quad \Rightarrow \quad \gamma = 90º$

Sol.4 A

$AB = (1, -3 - \alpha, 0)$

$CD = (3 - \beta, 2, -2)$

$AB \perp CD$

$(3 - \beta) + 2(-3 - \alpha) + 0 = 0$

$\beta + 2\alpha + 3 = 0$

Sol.5 D

$(xy + yz) = 0$

$x + z = 0$ and $y = 0$

Two perpendicular plane.

Sol.6 A

Normal vector of plane

$= (2 - 3, -1 -4, 5 + 1) = (-1, -5, 6)$

Equation of plane

$-x - 5y + 6z = k$

passes through $(2, -3, 1)$

$-2 + 15 + 6 = k \Rightarrow k = 19$

$-x - 5y + 6z = 19$

$x + 5y - 6z + 19 = 0$

Sol.7 A

$x + 2y + 2z = 5 \qquad \vec{n}_1 = (1, 2, 2)$

$3x + 3y + 2z = 8 \qquad \vec{n}_2 = (3, 3, 2)$

Normal vector of plane $= \vec{n}_1 \times \vec{n}_1$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 3 & 2 \end{vmatrix} = -2\hat{i} + 4\hat{j} + 3\hat{k}$$

Equation of plane

$-2x + 4y - 3z = k$

passing through $(1, -3, -2)$

$k = -8$

$-2x + 4y - 3z = -8$

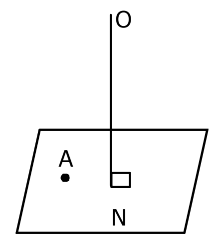
$2x - 4y + 8z - 8 = 0$

Sol.8 A

Let N be foot of poerpendicular $= (\alpha, \beta, \gamma)$

$N(\alpha, \beta, \gamma)$

A (1, 2, 3)



Equation of plane willk be

$\alpha x + \beta y + \gamma z = k$

passing through (1, 2, 3)

$\Rightarrow k = \alpha + 2\beta + 3\gamma$

$\alpha x + \beta y + \gamma z = \alpha + 2\beta + 3\gamma$

this plane passes through (α, β, γ) also

$\alpha^2 + \beta^2 + \gamma^2 = \alpha + 2\beta + 3\gamma$

$x^2 + y^2 + z^3 - x - 2y - 3z = 0$

Sol.9 B

$N(\alpha, \beta, \gamma)$

$3x - 2y - z = 9$

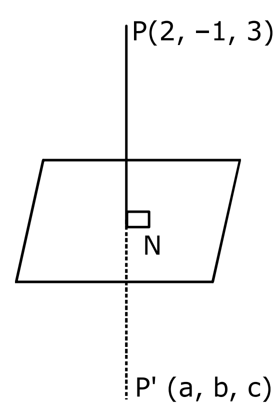
$$\frac{\alpha - 2}{3} = \frac{\beta + 1}{-2} = \frac{\gamma - 3}{-1} = \lambda$$

$\alpha = 3\lambda + 2, \beta = -2\lambda - 1, \gamma = -\lambda + 3$

N point lies on the plane

$3(3\lambda + 2) - 2(-2\lambda + 1) - (-\lambda + 3) = 9$

$\Rightarrow \lambda = \dfrac{2}{7}$



$N\left(\dfrac{20}{7}, \dfrac{-11}{7}, \dfrac{19}{7}\right)$

$N = \dfrac{P + P'}{2} \Rightarrow P^1 = 2N - P$

$\Rightarrow P^1\left(\dfrac{26}{7}, \dfrac{-15}{7}, \dfrac{17}{7}\right)$

Sol.10 D

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$$

Use pases through P(2, −1, 2)
point P
So P_0I of line and plane is P (2, −1, 2)
(−1, −5, −10) so PQ = 13

Sol.11 A

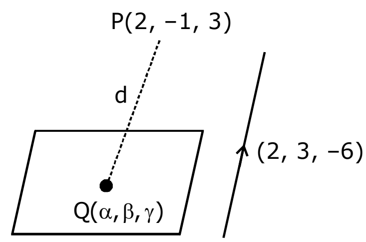
$$\frac{\alpha-1}{2} = \frac{\beta+2}{3} = \frac{\gamma-3}{-6} = \lambda$$

$\alpha = 2\lambda + 1$, $b = 3l - 2$, $\gamma = -6\lambda + 3$
(α, β, γ) lie on the plane x + y + z= 5

$$\Rightarrow \lambda = \frac{1}{7}$$

$$Q\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$$

d = PQ = 1



Sol.12 D

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

$$\& \ \frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{3}$$

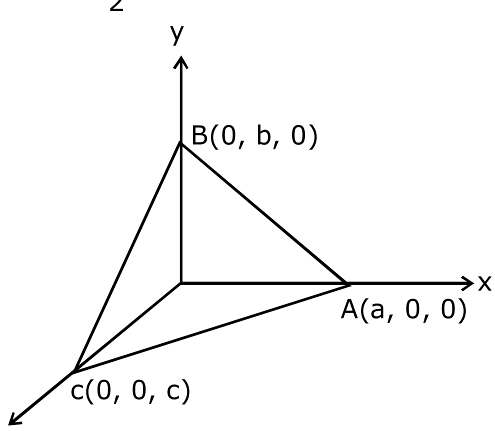
Both lines poasing through same point (1, 2, 3) that they intersect each other at point P.

$$\text{Angle } \cos\theta = \frac{(1.2)+(2.2)+(3.(-2))}{\sqrt{1+4+9}\sqrt{4+4+4}} = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

Sol.13 A

$$\text{Area} = \frac{1}{2}\ |\ \vec{AB} \times \vec{AC}\ |$$



$$= \frac{1}{2}\ |(-a, b, 0) \times (-a, 0, c)|$$

$$= \frac{1}{2}\ \sqrt{a^2b^2 + b^2c^2 + c^2a^2}$$

Sol.14 B

Let Point P (α, β, γ)
Given that
$(\alpha - 1^2) + (\alpha + 1)^2 + (\beta - 1) + (\beta + 1)^2 + (\gamma - 1)^2 + (\gamma + 1)^2 = 10$
$2\alpha^2 + 2\beta^2 + 2\gamma^2 + 6 = 0$
$\alpha^2 + \beta^2 + \gamma^2 = 2 \ \Rightarrow\ x^2 + y^2 + z^2 = 2$

Sol.15 A

Let the Eq^n of plane

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$$

passes through (a, b, c)

$$\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 1$$

common point will be (α, β, γ)
so locus

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$$

Sol.16 A

Let the equation of planes

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \ \& \ \frac{x}{a_1} + \frac{y}{b_1} + \frac{z}{c_1} = 1$$

perpendicular distance from orign will be same
$p_1 = p_2$

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right| = \left|\frac{-1}{\sqrt{\frac{1}{a_1^2}+\frac{1}{b_1^2}+\frac{1}{c_1^2}}}\right|$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}$$

Sol.17 B

$$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda \quad ....(1)$$

$$\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \mu \; ....(2)$$

Variable point on line (1) & (2)

$(3\lambda+1, \lambda+2, 2\lambda+3)$ & $(\mu+3, 2\mu+1, 3\mu-2)$

$3\lambda + 1 = \mu + 3$

$\lambda + 2 = 2\mu + 1$

$2\lambda + 3 = 3\mu + 2$

$2\lambda + 3 = 3\mu + 2$

$2\lambda + 3 = 3\mu + 2$

By solving $\lambda = 1$, $\mu = 1$

Intersection point (4, 3, 5)

Equation of plane

$4x + 3y + 5z = k$

passes through (4, 3, 5) $\Rightarrow$ k = 50

$4x + 3y + 5z = 50$

Sol.18 D

$2x - y + z = 6$ $\vec{n}_1 = (2, -1, 1)$

normal vector of other plane

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & -1 & 0 \end{vmatrix} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\text{angle } \cos\theta = \left| \frac{\vec{n}_1 . \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} \right| = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Sol.19 A

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -5\hat{i} + 5\hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ a & 1 & -1 \end{vmatrix} = -2\hat{i} + (2 + 3a)j + (2 + a)\hat{k}$$

p (0, –5, –3) ; R(0, –1/5, –3/5)

For compaire lines

$[\overrightarrow{PQ} \quad \vec{n}_1 \quad \vec{n}_2] = 0 \Rightarrow a = -2$

Sol.20 D

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} = \lambda \Rightarrow \text{point } (\lambda, 2\lambda, 3\lambda)$$

$$\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4} = M$$

$\Rightarrow$ Point (3M + 1, –M + 2, 4M + 3)

$$\frac{x+k}{3} = \frac{y-1}{2} = \frac{z-2}{h} = t$$

$\Rightarrow$ Point (3t – k, 2t + 1, ht + 2)

If all three lines are concurrent

$\lambda = 3\mu + 1$; $2\lambda = -\mu + 2$; $3\lambda = 4\mu + 3$

$\lambda = 1 \Rightarrow \mu = 1$

$3t - k = 1$; $2t + 1 = 2 \Rightarrow k = \frac{1}{2} \Rightarrow t = \frac{1}{2}$

$ht + 2 = 3$

$ht = 1 \Rightarrow h = 2$

Sol.21 A

A (2 –x, 2, 2) B (2, 2 – y, 2) C (2, 2, 2 – z) D(1, 1, 1)

$\overrightarrow{AB} = (x, -y, 0)$, $\overrightarrow{AC} = (x, 0, -2)$,

$\overrightarrow{AD} = (x - 1, -1, -1)$

If A, B, C, D are coplanar points then

$[\overrightarrow{AB} \;\; \overrightarrow{AC} \;\; \overrightarrow{AD}] = 0$

$$\begin{vmatrix} x & -y & 0 \\ x & 0 & -2 \\ x-1 & -1 & -1 \end{vmatrix} = 0 \quad \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

Sol.22 B

$|\overrightarrow{AC}| = 2$

$|\vec{a}| = |\vec{b}| = |\vec{a} - \vec{b}| = 4\sqrt{2}$

$|\vec{a} - \vec{c}| = 2$

$$\cos\theta = \frac{\left(\frac{\vec{b}}{2} - \vec{a}\right)\left(\frac{\vec{b}+\vec{c}}{2}\right)}{\left|\frac{\vec{b}}{2} - \vec{a}\right|\left|\frac{\vec{b}+\vec{c}}{2}\right|} = \frac{(\vec{b} - 2\vec{a}).(\vec{b}+\vec{c})}{|\vec{b} - 2\vec{a}|.|\vec{b}+\vec{c}|} = \frac{1}{\sqrt{2}}$$

Sol.23 A

A (a, b, c) B(a′, b′, c′)

Line $\overrightarrow{AB}$ = (a, b, c) + λ (a′ –a, b′ – b, c′ – c)
= (a + λa′, b + λb′, c + λc′) – λ(a, b, c)
It will passes through origin when
a + λa′ = b + λb′ = c + λc′ =0

$\Rightarrow \frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$

Sol.24 D

$|\overrightarrow{AC}| = 2$; $|\vec{a}| = |\vec{b}| = |\vec{a} - \vec{b}| = 4\sqrt{2}$

$|\vec{a} - \vec{b}| = 2$

$$\cos\theta = \frac{\left(\frac{\vec{b}}{2} - \vec{a}\right).\left(\frac{\vec{b}+\vec{c}}{2}\right)}{\left|\frac{\vec{b}}{2} - \vec{a}\right|\left|\frac{\vec{b}+\vec{c}}{2}\right|}$$

$$= \frac{(\vec{b} - 2\vec{a}).(\vec{b}+\vec{c})}{|\vec{b} - 2\vec{a}||\vec{b}-\vec{c}|}$$

put all the values $\cos\theta = \frac{1}{\sqrt{2}}$

Sol.25 A

Assume P is centroid

Sol.26 A

Direction of line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 3\hat{i} - 3\hat{j} + 3\hat{k}$

O.D. (x-axis) = $\frac{3}{\sqrt{a+a+a}} = \frac{1}{\sqrt{3}}$

Sol.27 D

$\ell = \cos\alpha = \frac{1}{\sqrt{2}}$

$\mu = \cos\beta = \frac{1}{\sqrt{2}}$

$\ell^2 + m^2 + n^2 = 1$

$n = 0 \Rightarrow \cos\gamma = 0 \quad \Rightarrow \gamma = \frac{\pi}{2}$

Sol.28 D

Direction of line = (1, 2, 2)

normal vector of plane = (2, –1, $\sqrt{\lambda}$)

$\sin\theta = \frac{2-2+2\sqrt{\lambda}}{\sqrt{1+4+4}\sqrt{4+1+\lambda}} = \frac{1}{3}$

$4\lambda = 5 + \lambda$

$\lambda = \frac{5}{3}$

Sol.29 C

$\cos^2\theta + \cos^2\beta + \cos^2\theta = 1$
$2\cos^2\theta = 1 - \cos^2\beta = \sin^2\beta$
$2\cos^2\theta = 3\sin^2\theta = 3 - 3\cos^2\theta$
$\cos^2\theta = 3/5$

Sol.30 C

2x + y + 2z = 8 ....(1)

2x + y + 2z = $-\frac{5}{2}$ ....(2)

Distance = $\frac{8+\frac{5}{2}}{\sqrt{4+1+4}} = \frac{21}{2\times3} = \frac{7}{2}$

Sol.31 B

x = y + a = z ....(1)
x + a = 2y = 2z ....(2)
we have option (B) & (C)
but ifwe look at option B
it will satisfy the given equation

Sol.32 A

Angle between two faces is equal to the angle between the normals $\vec{n}_1$ and $\vec{n}_2$.

$\vec{n}_1 \rightarrow$ normal of OAB

$\vec{n}_2$ = normal of ABC

$\vec{n}_1 = \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$

$= 5\hat{i} - \hat{j} - 3\hat{k}$ ...(1)

$\vec{n}_2 = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix}$

$= \hat{i} - 5\hat{j} - 3\hat{k}$ ...(2)

$$\cos\theta = \frac{\vec{n_1}.\vec{n_2}}{|\vec{n_2}||\vec{n_2}|} = \frac{19}{35} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

Sol.33 C

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k};$$

$$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$$

A(2, 3, 4) B (1, 4, 5)
D.R. (1, 1, –k) D.R. (k, 2, 1)

$$\text{Coplanar then} = \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ k = 0 or k = –3

Sol.34 D

x = ay + b, z = cy+ d
and x = a'y + b', z = c'h + d'

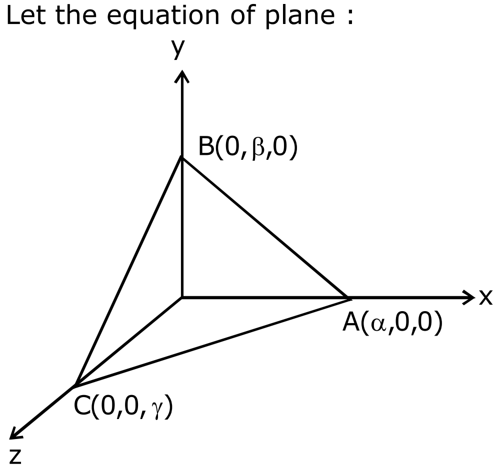
$$\frac{x-b}{a} = y = \frac{z-d}{c}$$

and $\frac{x-b'}{a'} = y = \frac{z-d'}{c'}$

poerpendicular then
aa' + 1 + cc' = 0

Sol.35 C

Let the equation of plane :



$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1 \qquad ....(1)$$

$\frac{\alpha}{3} = a \qquad \Rightarrow \alpha = 3a$

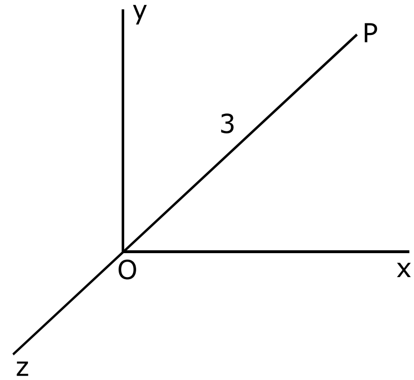
$\frac{\beta}{3} = b \qquad \Rightarrow \alpha = 3a$

$\frac{\gamma}{3} = c \qquad \Rightarrow \gamma = 3c$

$$\frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$$

Sol.36 A

D.R. of OP = (1, –2, –2)



D.C. of OP = $\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$

Vector $\vec{OP} = |\vec{OP}| \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$

= (1, –2, –2)

Sol.37 B

$\vec{a} = (1, 5, -3)$

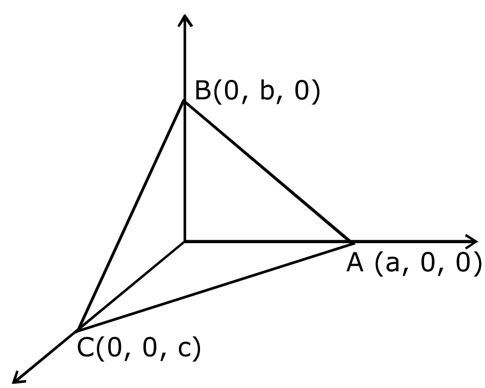
$\vec{b} = (-1, 8, 4)$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

Sol.38 A

Ley the equation of plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$



given that $p = \dfrac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$

or $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$ ....(1)

Let centroid (u, v, w)

$u = \frac{1}{4} \quad \Rightarrow a = 4u$

$v = \frac{b}{4} \quad \Rightarrow b = 4v$

$w = \frac{c}{4} \quad \Rightarrow c = 4w$

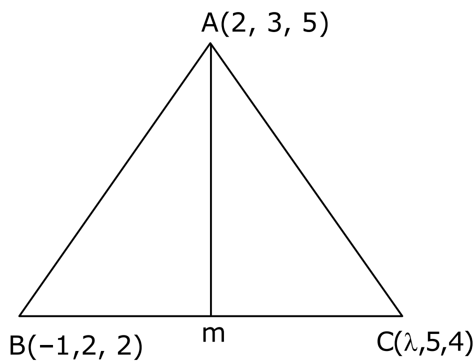
$$\frac{1}{16u^2} + \frac{1}{16v^2} + \frac{1}{16w^2} = \frac{1}{p^2}$$

$$\frac{1}{u^2} + \frac{1}{v^2} + \frac{1}{w^2} = \frac{16}{p^2}$$

$u^{-2} + v^{-2} + z^{-2} = 16p^{-2}$

Sol.39 C

A (2, 3, 5) B(−1, 2, 2) C(λ, 5, 4)



$m\left(\frac{\lambda-1}{2}, \frac{7}{2}, \frac{\mu+2}{2}\right)$

D.R> of median through A :

$$\left(\frac{\lambda-1}{2} - 2\,\frac{7}{2} - 3, \frac{\mu+2}{2} - 5\right)$$

$$\left(\frac{\lambda-5}{2}, \frac{1}{2}, \frac{\mu-8}{2}\right)$$

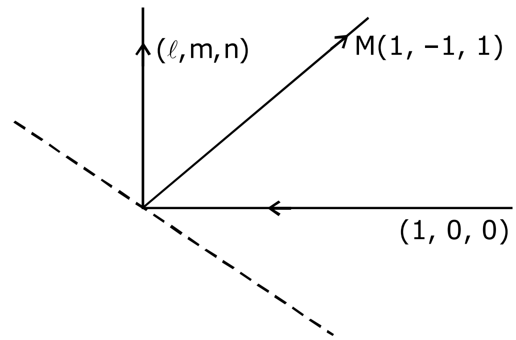
As thje median through A is equally inclined to He axis

∴ D.R.'s will be and equal to k.

$$\frac{\frac{\lambda-5}{2}}{k} = \frac{1}{2k} = \frac{\frac{\mu-8}{2}}{k} \quad \Rightarrow \lambda = 6 \text{ and } \mu = 9$$

Sol.40 D

The DC's of incident RAy arew (1, 0, 0). Let the D.C's of reflectd ray be (λ, m, n)



⇒ The D.R.'s of the normal to polane of mirror is (l – 1, m, n)

$$\frac{\ell-1}{1} = \frac{m}{-1} = \frac{n}{1}$$

$\ell = \lambda + 1,\ m = -\lambda,\ n = \lambda$

$\ell^2 + m^2 + n^2 = 1$

$(\lambda + 1)^2 + \lambda^2 + \lambda^2 = 1$

$3\lambda^2 + 2\lambda = 0$

$\lambda = -2/3$

D.C's of reflected Ray $\left(\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$

or $\left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

Sol.41 B

$$\text{dir}^n \text{ of line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{vmatrix} = -2\hat{i} + \hat{k}$$

DR′ & = (−2, 0, 1)

$(\vec{n}_1 \times \vec{n}_2) \times \hat{k} = (-2\hat{i} + \hat{k}) \times \hat{k} = 2\hat{j}$

⇒ distance = 2

Sol.42 C

$$\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$$

$(3\lambda + 2, 2\lambda - 1, 1 - \lambda)$

$z = 0 \quad \Rightarrow \quad \lambda = 1$

$xy = c^2$

$(3\lambda + 2)(2\lambda - 1) = c^2$

put $\lambda = 1 \Rightarrow c^2 = 5 \Rightarrow c = \pm\sqrt{5}$

Sol.43 C

Distance = $\sqrt{x^2 + y^2 + z^2}$

= $\sqrt{(2t)^2 + (4t)^2 + (4t)^2}$

= 6t t = 10

Distance = 60 km

Sol.44 B

Let the point P(x, y, z)

Asking minimum value of OP^2

$\Rightarrow$ $\perp^r$ distance of origin from plane

$$d = \left|\frac{P}{\sqrt{a^2 + b^2 + c^2}}\right| \Rightarrow d^2 = \frac{P^2}{\Sigma a^2}$$

Sol.45 B

Since three lines are mutually perpendicular

$\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$; $\ell_2\ell_3 + m_2m_3 + n_2n_3 = 0$

$\ell_3\ell_1 + m_3m_1 + n_3n_1 = 0$

Also $\ell_1^2 + m_1^2 + n_1^2 = 1$; $\ell_2^2 + m_2^2 + n_2^2 = 1$;

$(\ell_1 + \ell_2 + \ell_3)^2 + (m_1 + n_2 + m_3)^2 + (n_1 + n_2 + n_3)^2$

$= (\Sigma\ell_1^2 + \Sigma\ell_2^2 + \Sigma\ell_1\ell_3^2 + 2\Sigma\ell_1\ell_2 + 2\Sigma\ell_2\ell_3 + 2\Sigma\ell_3\ell_1) = 3$

$\Rightarrow (\ell_1 + \ell_2 + \ell_3)^2 + (m_1 + m_2 + m_3)^2 + (n_1 + n_2 + m_3)^2 = 3$

Hence direction cosines of OP are

$$\left(\frac{\ell_1 + \ell_2 + \ell_3}{\sqrt{3}}, \frac{m_1 + m_2 + m_3}{\sqrt{3}}, \frac{n_1 + n_2 + n_3}{\sqrt{3}}\right)$$

Sol.46 C

Equation of lines :

$$\frac{x-2}{3-2} = \frac{y+3}{-4+3} = \frac{z-1}{-5-1}$$

$$\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = \frac{z-1}{-6} = \lambda$$

Points $(\lambda + 2, -\lambda - 3, -6\lambda + 1)$

Point will be on given plane

$2(\lambda + 2) + (-\lambda - 3) + (-6\lambda + 1) = 7$

$\Rightarrow \lambda = -1$

Intersection point (1, –2, 7)

Sol.47 A

Direction ratio's of line = (–2, 1, 2)

Direction cosine's = $\left(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

$\cos\theta = \frac{-2}{3}$, $\cos\theta_2 = \frac{1}{3}$; $\cos\theta_3 = \frac{2}{3}$

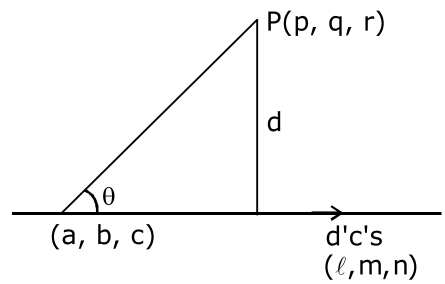
$\cos2\theta_1 + \cos2\theta_2 + \cos2\theta_3$

$= 2\,[\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3] - 3$

$= 2\left[\frac{4}{3} + \frac{1}{3} + \frac{4}{3}\right] - 3 = -1$

Sol.48 A

$\frac{x-a}{\ell} = \frac{y-b}{m} = \frac{z-c}{n}$ Point (p, q, r)



Let $\vec{r_1} = (p - a)\,\hat{i} + (q - b)\,\hat{j} + (r - c)\,\hat{j}$

$\vec{r_2} = \ell\,\hat{i} + m\,\hat{j} + n\,\hat{k}$

$$\cos\theta = \frac{\vec{r_1}\,.\,\vec{r_2}}{|\vec{r_1}|\,|\vec{r_2}|}$$

also $d = |\vec{r_1}|\sin\theta$

$d^2 = |\vec{r_1}|^2 \sin^2\theta$

$= |\vec{r_1}|^2 (1 - \cos^2\theta)$

$$= |\vec{r_1}|^2 \left[\frac{(\vec{r_1}\,.\,\vec{r_2})^2}{|\vec{r_1}|^2 |\vec{r_2}|^2}\right]$$

$d^2 = |\vec{r_1}|^2 - (\vec{r_1}\cdot\vec{r_2})^2$

$= [(P - a)^2 + (q - b)^2 + (r - c)^2] - [\ell\,(p - a) + m(q - b) + n(r - c)]^2$